

this difficulty can be lessened by taking 500 days instead of 1000 as the second unit. But this will detract from simplicity of form.

A yet more serious drawback to the theoretical investigator is that the fundamental epochs usually adopted in astronomy, and for which the elements must be found, do not correspond to any power of 10 in the days of the Julian period. A complete transformation of the elements is therefore required to form the numbers on which the tables are based. If, therefore, multiples of 500 or 1000 days are used instead of years, I should prefer to count them back from 1900·0, thus gaining all the advantages of the Julian period without any other disadvantage than that of non-correspondence with the eclipse and other tables of Oppolzer.

The reduction of such a system to the ordinary calendar may be made a very simple matter. It seems to me, therefore, that the maximum of advantage will be reached by giving the fundamental arguments for cycles and periods based on multiples of 500 days before and after the fundamental epoch 1900 Jan. 0.

Probably the most convenient fundamental quantities to tabulate will be the longitude of the node, and the mean distances of the Moon and of its perigee from the node, all expressed in circumferential units. Then, whatever form the tables may be thrown into, we shall have the nearest approach to a simple, straight-ahead computation.

Finally, a serious problem is that of summing perhaps 100 periodic terms with coefficients not differing greatly from $0''\cdot01$. I have devised a machine for this purpose, the description of which must form the subject of another publication.

An Example of Professor Karl Pearson's Calculation of Correlation in the case of the Periodic Inequalities of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following note is written with a twofold purpose. Firstly, it is hoped that an indication of some value has been obtained with regard to the features of "long-period" variability; and secondly, the opportunity is taken to write out in full a simple example of the calculation of "correlation" between quantities by the methods of Professor Karl Pearson.

In the *M.N.* for March last (p. 416) Professor Pearson himself gave an admirable summary of methods; but he naturally did not repeat the elementary working which has become so familiar to him, and has been given often before in other connections. There are doubtless many to whom this working is already familiar; but there are certainly many others who do not know it and who might use it if they had an astronomical example readily accessible. In these busy days many people have not the leisure to search for references in scientific literature outside their own subject.

2. In support of my view that there is need for an example, I may cite an illustration shown me by Professor E. C. Pickering (see *Observatory*, March 1905, p. 153).

Suppose we have the following observed values of A and B:—

A	.	.	.	5	6	7	6	7	8	7	8	9
B	.	.	.	6	6	6	7	7	7	8	8	8

First group according to B, in sets of three for which B has the same value.

Then when $B = 6 \quad 7 \quad 8$

Mean $A = 6 \quad 7 \quad 8$

Therefore $A = B$.

Secondly, group according to A.

Then when $A = 5 \quad 6 \quad 7 \quad 8 \quad 9$

Mean $B = 6 \quad 6\frac{1}{2} \quad 7 \quad 7\frac{1}{2} \quad 8$

Therefore $(A - 7) = 2(B - 7)$.

Now, neither Professor Pickering nor myself was prepared at that time to deal with this situation by a definite process; and yet this is just an elementary case of the kind which Professor Pearson's methods were devised to meet.

This example is worked out below, and it is shown that the numerical measure of the correlation is

$$r = .08 \pm .22$$

or almost nothing at all: so that, in spite of appearances to the contrary, we are not entitled to assume any relationship between A and B. Putting it in another way, one proposed relation ($A = B$) is as good as another ($A - 7 = 2(B - 7)$).

3. A point of detail may be mentioned here. In much statistical work, a large number of figures are used. Thus we get such statements* as

$$r = \frac{1262.51}{2870 \times 1.13637 \times 1.52135} = .25445.$$

The probable error of r being about $\pm .01$, some of these figures are superfluous; and in what follows fewer figures are used. This, however, represents a personal view which is, I find, not generally approved by other workers.

4. The particular example selected for treatment is the discussion of the elements of maximum given by Chandler for long-period variables. The following particulars are taken from

* *Frequency Curves and Correlation*, by W. P. Elderton, p. 119: an excellent little book, from which much is to be learned.

his "Revision of Elements of Third Catalogue" (*Astronomical Journal*, No. 553):—

Star.	Epoch.	Elements of Maximum.		Inequalities.
R Androm.	2400141	+	410.3 E	+ 30 sin (12° E + 90°)
S Cassiop.	2401603	+	610.5 E	+ 37 sin (15° E + 59°)

and so on. The mean period of S Cassiop. is 610.5 days; but there is an inequality which displaces the maximum 37 days one way, and 12 periods later (since $180^\circ/15^\circ = 12$) the displacement is 37 days in the other direction. At these times the period is about the mean, but midway between such times it is longer and shorter. In the "Revision" above mentioned, Chandler gives such inequalities for 37 stars, as shown in Table I. We shall for the present only consider the period P, the coefficient C, and the argument A; thus for S Cassiop. $P=610.5$, $C=37$, and $A=15^\circ$. The epoch 59° does not concern us at present.

TABLE I.

Star's Name.	No.	P	C	A	Phase.	M - m	αP
R Androm.	112	410	30	12	218	120	- 170
S Cassiop.	432	610	37	15	148	280	- 50
S Piscium	434	404	18	10	57	163	- 78
R Arietis	782	187	7	5	292	92	- 3
o Ceti	806	332	18	5	...	125	- 82
R Persei	1222	210	15	8	224	96	- 18
R Aurigæ	1855	459	19	12	320	235	+ 11
R Lyncis	2478	379	14	15	249	186	- 7
R Gemin.	2528	370	35	6	121	121	- 128
S Can. Min.	2684	330	20	12	120	164	- 2
R Cancri	2946	362	60	6	341	125	- 112
R Carinæ	3418	310	25	9	289	136	- 38
R Leo Min.	3477	371	20	10	11	165	- 41
R Urs. Maj.	3825	302	11	8	63	110	- 82
T Urs. Maj.	4511	257	20	9	203	108	- 42
R Virginis	4521	145	20	2	103	68	- 9
S Urs. Maj.	4557	226	35	5	309	108	- 10
S Boötis	5157	270	60	4	87	132	- 6
R Camelop.	5190	270	65	4	244	142	+ 14
R Boötis	5237	223	9	9	251	102	- 20

TABLE I.—*continued.*

Star's Name.	No.	P	C	A	Phase.	M - m	αP
S Serpentis	5501	369	116	4	122°
S Coronæ	5504	361	8	12	81	120	- 121
R Serpentis	5677	357	35	4	109	151	- 55
R Herculis	5770	318	17	10	29
W Herculis	5950	280	26	13	288	128	- 24
S Herculis	6044	308	35	9	239	152	- 4
T Herculis	6512	165	10	5	152	79	- 7
R Sagittarii	6905	269	18	10	18	138	+ 7
S Sagittarii	6921	231	15	10	167	102	- 27
R Delphini	7261	284	26	9	266	130	- 24
U Capric	7455	203	20	5	67
T Aquarii	7468	203	8	7	357	88	- 27
R Vulpec.	7560	137	18	4	127	62	- 13
X Capric.	7577	218	20	10	98	117	+ 16
R Pegasi	8290	378	60	8	71	172	- 34
R Aquarii	8512	387	35	10	97
R Cassiop.	8600	432	32	9	233	182	- 68

5. The first two columns of Table I. give the star's name and its number on Chandler's system. The third column, P, gives the period in days; C is the coefficient of the periodic inequality in days, and A the coefficient of E within the bracket. The explanation of the last three columns will be given later.

6. Now the example selected to be given in full is that of the relation between P and A. To determine whether there is such a relation, we could, of course, proceed in the ordinary way to group together stars with nearly the same P, and take the mean values of A for them. This would give us a fair indication, but no notion of its probable error. Professor Pearson's procedure is not much longer; it includes the ordinary procedure, and it gives us a definite measure of the probability of the relation being real.

7. We first form a "correlation table" as in Table II.

[TABLE

TABLE II.
Correlation Table for P and A.

P in days. \ A =	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	Sums
601 to 650														1 ₃₆	1
551 to 600														0 ₈₀	0
501 to 550														0 ₂₄	0
451 to 500								0	0 ₃	0 ₆	1 ₉	0	0	0 ₁₈	1
401 to 450								1	1 ₂	0 ₄	1 ₆	0	0 ₈	0 ₁₀	3
351 to 400			2 ₅	0 ₄	2 ₃	0 ₂	1 ₁	0	2 ₁	0 ₂	1 ₃	0	0 ₄	1 ₅	9
301 to 350			0	1	0	0	1	2	1	0	1	0	0	0	6
251 to 300			2 ₅	0 ₄	0 ₃	0 ₂	0 ₁	2 ₁	1 ₁	0 ₂	0 ₃	1 ₄	0 ₅	0 ₆	6
201 to 250			0 ₁₀	2 ₈	0 ₆	1 ₄	1 ₂	1	2 ₂	0 ₄	0 ₆	0 ₈	0 ₁₀	0 ₁₂	7
151 to 200			0 ₁₅	2 ₁₂	0 ₉	0 ₆	0 ₃								2
101 to 150	1 ₂₈	0 ₂₄	1 ₂₀	0 ₁₆	0 ₁₂	0 ₈	0 ₄								2
Sums	1	0	5	5	2	1	3	6	7	0	4	1	0	2	37

A	Sum.	Factor.	$\nu_1 =$ Product.	$\nu_2 =$ Product.
2	1	-7	-7	49
3	0	-6	0	0
4	5	-5	-25	125
5	5	-4	-20	80
6	2	-3	-6	18
7	1	-2	-2	4
8	3	-1	-3	3
9	6	0	0	0
10	7	+1	+7	7
11	0	+2	0	0
12	4	+3	+12	36
13	1	+4	+4	16
14	0	+5	0	0
15	2	+6	+12	72
Sums	37	...	-28	410
Means (ν_1 and ν_2)			-·84	+11·08
$\sigma_A = \sqrt{\nu_2 - \nu_1^2} = \sqrt{10 \cdot 37} = 3 \cdot 22$				

P	Sum.	Factor.	$\nu_1 =$ Product.	$\nu_2 =$ Product.
125	2	-4	-8	32
175	2	-3	-6	18
225	7	-2	-14	28
275	6	-1	-6	6
325	6	0	0	0
375	9	+1	+9	9
425	3	+2	+6	12
475	1	+3	+3	9
525	0	+4	0	0
575	0	+5	0	0
625	1	+6	+6	36
Sums	37	...	-10	150
Means (ν_1 and ν_2)			-·27	4·05
$\sigma_P = \sqrt{\nu_2 - \nu_1^2} = \sqrt{3 \cdot 98} = 2 \cdot 00$				

Products S'(AP)
= 17 + 64
+ 104 = 9
= +142
37(-·84)(-·27) = + 9
Difference = 133
 $r = \frac{133}{37 \times 3 \cdot 22 \times 2 \cdot 00}$
= 0·56

8. This requires little explanation. We divide the periods into groups of 50 days as shown in the first column, and the number of periods in each group is shown in the final column. For the values of A no collection in groups is necessary, successive degrees being shown in the top line, and the number of corresponding cases in the bottom line. In the body of the table are given, in large figures, the number of cases corresponding to a definite degree for each group of periods. The small figures will be referred to presently.

9. The thick lines indicate the groups which contain approximately the mean of each co-ordinate. They are chiefly for convenience in working (to keep the numbers small), and if a wrong group is taken no real harm is done.

10. The figures below the table represent the whole working. We first form ν_1 and ν_2 for each co-ordinate by a process which is tolerably clear to inspection. For A write down the (top line for convenience only—not used in working; and the) bottom line of the table. Follow with a sequence of units, with 0 at the place of the adopted mean. Multiply the two columns; and again multiply the last two columns. We thus get $n\nu_1$ and $n\nu_2$, where n is the number of observations, and ν_1 , ν_2 two quantities required. Dividing by n , we get ν_1 and ν_2 , and finally

$$\sigma_A = (\nu_2 - \nu_1^2)^{\frac{1}{2}} = 3.22.$$

[If greater accuracy is required, we should subtract also "Sheppard's correction" of $\frac{1}{12}$ from ν_2 before taking the square root.]

11. The same procedure with the columns gives us $\sigma_P = 2.00$.

12. We now form the product $S(AP)$. The number in each box is to be multiplied by the little number shown in the corner of the box, which will be seen to be the product xy , where x represents the number of boxes to the right, and y the number up, from the adopted mean. The products in the four main divisions of the table are shown diagrammatically, and from them we get

$$S'(AP) = 142.$$

From this we subtract $n\nu_1\nu_1^1$, where n is the number of observations, and ν_1 , ν_1^1 have been found; and thus

$$S(AP) = S'(AP) - n\nu_1\nu_1^1 = 142 - 9 = 133.$$

Then the "correlation" r is

$$r = \frac{S(AP)}{n \times \sigma_A \times \sigma_P} = 0.56.$$

13. This is quite a respectable value for r —anything over 0.5 is worth serious attention. But to complete our information we want the probable error of r , which is given by the formula

$$p.e. = .67(1 - r^2)/\sqrt{n},$$

which can be taken from the following small table (Table III.).

For the present case we have $n = 37$, $r = 0.56$, and thus probable error $= \pm .08$.

TABLE III.

Probable error of r .

r \backslash n	.1	.2	.3	.4	.5	.6	.7	.8	.9
10	.208	.202	.193	.177	.158	.136	.107	.076	.041
20	.148	.143	.137	.125	.112	.096	.076	.054	.029
30	.121	.117	.112	.102	.091	.079	.062	.044	.024
40	.104	.101	.096	.088	.079	.068	.054	.038	.021
50	.093	.090	.086	.079	.070	.061	.048	.034	.018
60	.085	.083	.079	.072	.064	.056	.044	.031	.017
70	.079	.076	.073	.067	.059	.051	.040	.029	.015
80	.074	.072	.068	.063	.056	.048	.038	.027	.015
90	.069	.067	.064	.059	.053	.045	.036	.025	.014
100	.066	.064	.061	.056	.050	.043	.034	.024	.013

14. Coming now to the relation between P and A , the mean value of A is got by adding ν_1 to our adopted mean, *i.e.* it is $9^\circ - 0^\circ.84 = 8^\circ.16$; and the mean value of P is $325 - 0.27 \times 50 = 311.5$ days, the unit adopted in the table being 50 days, by which therefore we multiply ν_1 .

Assuming a linear relation between A and P , there are two appropriate forms for it, *viz.*

$$(A - 8^\circ.16) = r \cdot \frac{\sigma_A}{\sigma_P} \left(\frac{P - 311}{50} \right) \text{ and } \left(\frac{P - 311}{50} \right) = r \cdot \frac{\sigma_P}{\sigma_A} (A - 8^\circ.16),$$

and those new to correlation methods will notice with surprise that these are not convertible. To save repeating explanations already given in detail elsewhere, we need only remark that to find the most probable value of A when P is given is not the same as to find the most probable value of P when A is given. There is the same *kind* of difference (the analogy must not be taken too seriously) as between finding from the fact that a man is 25 the date of his probable death, and finding from the date of his death the probable date of his being 25: the latter case is clearly complicated by the possibility that he may never have reached 25 at all.

15. In astronomical examples it will frequently happen that one of the variables is better determined than the other, and hence one of the two alternative forms is indicated as preferable. Thus in our case P is well known; most of the periods are known within a day or two, some perhaps to fractions of a day; and since our unit in Table II. was 50 days, the precision is less than one-tenth of the unit. But A may be several degrees wrong, *i.e.* several

whole units. Hence it is natural to determine A from P, and the appropriate equation is

$$(A - 8^{\circ}16) = 0.56 \times \frac{3.22(P - 311)}{50}$$

or $A = 2^{\circ}6 + .018P,$

P being now measured in *days*, and not in units of 50 days. The most probable values of A corresponding to different periods are thus :

For P = 100 ^d	200 ^d	300 ^d	400 ^d	500 ^d	600 ^d
A = 4°·4	6°·2	8°·0	9°·8	11°·6	13°·4

16. This single example will suffice to show the method of finding *r* in such cases. There are many other points to be considered, *e.g.* how to deal with cases where the relation is not linear, or where one of the quantities is not measured but only described. But it seems probable that in astronomy the evaluation of *r* by the above simple process will be the point most often occurring.

17. But before leaving the more general topic of correlation for the particular application of it we have in view, it is a matter of some interest to return now to the simple example given in § 2, and work it out as below.

TABLE IV.
The example given in § 2.

A \ B	5	6	7	8	9	
6	1 ₂	1 ₁	1			3
7		1	1	1		3
8			1	1 ₁	1 ₂	3
	1	2	3	2	1	9

A	Sum.	Factor.	ν_1	ν_2	B	Sum.	Factor.	ν_1	ν_2	Product S'(AB)
5	1	-2	-2	4	6	3	-1	-3	3	
6	2	-1	-2	2	7	3	0	
7	3	0	8	3	+1	+3	3	
8	2	+1	+2	2	Sums	9	...	0	6	
9	1	+2	+2	4						Product S'(AB)
Sums	9	...	0	12						$\begin{array}{r l} +3 & 0 \\ \hline 0 & +3 \\ \hline & =6 \end{array}$

$\sigma_A = \sqrt{12} = 3.46$ $\sigma_B = \sqrt{6} = 2.45$

$r = \frac{6}{9 \times \sqrt{12} \times \sqrt{6}} = .08.$

This is a very low value, and on referring to Table III. we see that the probable error is greater than 0.2 : so that the correlation of the quantities A and B may be regarded as unsupported. And yet at first sight there is a fair appearance of relationship. This shows the advantage of having a numerical test.

18. We proceed now to discuss the interpretation of this and similar results. And in the first place it is an obvious suggestion to inquire whether C is also related to P. In a precisely similar manner, which need not be given in detail, we find the correlation for C and P to be

$$r = .30 \pm .10,$$

and for the probable relationship

$$(C - 27.2) = .064(P - 311)$$

or
$$C = 7.3 + .064P,$$

where C and P are measured in days. But the value of r and its probable error show that the probability of a relationship is here more doubtful. This is quite possibly due to the much greater uncertainty of the values of C, which is more difficult to determine than A, for reasons which those who have discussed variable star observations will readily appreciate.

With this formula the values of C would be:

P = 100 ^d	200 ^d	300 ^d	400 ^d	500 ^d	600 ^d
C = 14	20	26	33	39	46

19. Now, having obtained the suggestion of correlation and two formulæ for A and C, I examined in detail some individual cases which had come under notice and which are given below. It will be seen that there is good confirmatory evidence, so far as it goes, of these relationships.

S Cassiopeiæ (Ch. No. 432).

20. In *Mem. R.A.S.*, lv., p. lxix, the Rousdon observations are compared with Chandler's formula in his 3rd Catalogue:

$$610.5E + 50 \sin (10^\circ E + 50^\circ).$$

From observations $E = 17$ to 21 (about) a period of 630.7 days is derived, differing sensibly from 606.2 given by the formula. It is remarked: "To obtain [630.7 days] from the formula we must alter the periodic term in some way, and especially we must increase the coefficient, for with the coefficient 50 the greatest interval between two consecutive maxima is the greatest value of

$$610.5 + 50[\sin \theta - \sin (\theta + 10^\circ)],$$

which is $610.5 + 100 \sin 5^\circ$ or 619.2 days. But we cannot discuss this point without reference to other observations. Chandler used maxima in 1843 and 1863-93, and any new formula must thus

agree with the old one at these periods. Tentatively we may suggest some such modification as

$$240.1590 + 6.15E + 75 \sin (12^\circ E + 90^\circ)."$$

Chandler's revision gives

$$610.5E + 37 \sin (15^\circ E + 59^\circ).$$

The value of the periodic term deduced from the period 610 days by the formulæ found is

$$47 \sin (13^\circ.6 E + ?).$$

The coefficient given by the formula is thus in accord with Chandler's first thought, and lies between his revised value and the value suggested very tentatively by the Rousdon observations. The mean of the three different suggested values, 75, 50, and 37, is 54; and the fact that the smallest of these, 37, was used in deducing the formula accounts for part of the difference between the formula value 47 and this mean.

It is perhaps worth noting that if the suggestion of $C = 75$ (made quite independently of the present investigation) is adopted instead of Chandler's $C = 37$, the value of r is raised from .30 to .43.

The argument $13^\circ.6$ given by the formula is between the value suggested by the Rousdon observations and Chandler's revision.

R Ursæ Majoris (Ch. No. 3825).

21. In *Mem. R.A.S.*, lv., p. lxxvi, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$302.1E + 15 \sin (10^\circ E + 190^\circ).$$

It is shown that the Rousdon observations indicate an error of 150° in the periodic term: so that the coefficient of E should be 8° instead of 10° . Chandler's revision gives

$$302.1E + 11 \sin (8^\circ E + 238^\circ).$$

Pogson's observations of this star have since been very carefully discussed by Miss Blagg; and the outcome of this entirely independent discussion (which will shortly be published in *Mem. R.A.S.*, lviii., in the introduction to Pogson's observations) was to indicate a periodic term

$$40 \sin (8^\circ E + ?).$$

The formula gives for 302 days a term

$$27 \sin (8^\circ E + ?),$$

in which the coefficient is midway between Chandler's revision and the Pogson indication; and the argument in good accordance with both.

T Ursæ Majoris (Ch. No. 4511).

22. In *Mem. R.A.S.*, lv., p. lxxviii, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$257.2E + 20 \sin(9^\circ E + 90^\circ),$$

with the conclusion that the agreement is good. Chandler's revision gives no change. The correlation formulæ give for a period of 257 days a periodic term $24 \sin(7^\circ E + ?)$.

S Ursæ Majoris (Ch. No. 4557).

23. On p. lxxx the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$226.1E + 43 \sin(5^\circ.76 E + 181^\circ.5),$$

with the conclusion that "there is a fairly satisfactory accordance, though some correction to the formula would improve it."

Chandler's revision gives

$$226.5E + 35 \sin(5^\circ.4 E + 194^\circ).$$

The correlation formulæ give for a period of 226.5 days a periodic term $22 \sin(6^\circ.5 E + ?)$, which suggests that Chandler's diminution of the coefficient has not been carried far enough, though in the right direction.

S Cygni (Ch. No. 7220).

24. On p. lxxxix the Rousdon observations are compared with Chandler's formula

$$322.8E + 15 \sin(12^\circ E + 66^\circ).$$

It is remarked that the "periods at (the Rousdon) epoch would agree better if the coefficient of E in the periodic term were smaller, say 9° instead of 12° ."

In Chandler's revision the periodic term is replaced by a secular term

$$323E + 0.015E^2.$$

The correlation formulæ give for a period of 323 days

$$28 \sin(8^\circ E + ?),$$

so that the only suggestion made at the time of discussing the Rousdon observations is in the direction of better accordance with the formulæ.

R Sagittæ (Ch. No. 7257).

25. This star might have been included in Table I., but it was decided to draw the line as regards "long-period variables" at 100 days. The elements given in the revision are

$$70.56E + 6.5 \sin(2^\circ.25 + 47^\circ).$$

Extrapolating our formulæ, we get $C = 12$ and $A = 3^{\circ}9$. But if we decide that this star may be regarded as a long-period variable, we should include it in our table, and then the values of C and A would be found closer to those observed, since the star would have great weight. Indeed, the correlation for A and P is raised to $r = 0.64 \pm 0.07$, and the formula becomes

$$(A - 8^{\circ}1) = 1.01 \left(\frac{P - 305}{50} \right)$$

or
$$A = 1^{\circ}9 + 0.020 P,$$

which gives $A = 3^{\circ}3$ for $P = 70$.

The value of r for C and P is, however, not much improved, being raised from $.30$ to $.32$.

R Cassiopeiæ (Ch. No. 8600).

26. On p. xcii of the Rousdon Memoir, the observations are compared with Chandler's 3rd Catalogue formula,

$$429.5E + 25 \sin (15^{\circ} E + 0^{\circ}),$$

and it is remarked:—

"The corrections to period shown by the different columns of the Rousdon observations are so consistent that it is difficult to believe that the mean result can be so erroneous as the formula would make it. If the formula were altered to

$$431E + 30 \sin (12^{\circ} E + 9^{\circ})$$

[there would be a certain improvement]; but it is of course impossible to alter the formula definitively without discussing other observations."

Chandler's revision, which appeared after the above words were in type, gives

$$431.6E + 32 \sin (9^{\circ} E + 60^{\circ}).$$

The correlation formulæ give for the period 431 days a periodic term

$$35 \sin (10^{\circ}7 E + ?),$$

showing that the improvements suggested by the Rousdon observations were in the right direction, though not sufficient in magnitude; and that Chandler's revision is in good accord with the formula.

S Delphini (Ch. No. 7431).

27. Chandler gives no indication of a periodic term in his "revision," printing the period as 277.5 days. In discussing Baxendell's observations (before the present correlation work had been undertaken at all), Miss Blagg found clear indications of a periodic term at which a preliminary guess of $9 \sin (7\frac{1}{2}^{\circ} E + ?)$ was made, the 9 being mere guess-work, but the $7\frac{1}{2}^{\circ}$ being indicated with

fair precision. The correlation formulæ give for a period 277·5 days $A = 7\cdot6$, which is close to the value guessed. For the coefficient C the formula gives 25 days, which is a good deal larger than the guessed 9; but on *calculating* the coefficient from the separate corrections to epoch the coefficient came out 28·3, which accords well with the formula. Although the calculation is of a rough kind, it seems worth while giving it here to show the kind of accordance that is at present obtainable.

TABLE IV.
Baxendell's *S. Delphini*.

E	Observed Correction to Epoch.	Argument ($7\frac{1}{2}^\circ E + \text{const.}$)	Sin (Arg.).	Product.
I.	- 12 ^d	- 26°	- ·44	+ 5·3
II.	- 5	- 18½	- ·32	+ 1·6
III.	+ 3	- 11	- ·19	- 0·6
IV.	- 5	- 3½	- ·09	+ 0·5
V.	+ 11	+ 4	+ ·07	+ 0·8
VI.	+ 20	+ 11½	+ ·20	+ 0·4
VII.	+ 21	+ 19	+ ·33	+ 6·9
VIII.	+ 6	+ 26½	+ ·45	+ 2·7
IX.	0	+ 34	+ ·56	0·0
X.	+ 23	+ 41½	+ ·66	+ 15·2
XI.	+ 38	+ 49	+ ·76	+ 28·9
<hr/>				
XXIV.	+ 5	+ 146½	+ ·55	+ 2·8
XXV.	+ 3	+ 154	+ ·44	+ 1·3
XXVI.	+ 1	+ 161½	+ ·32	+ 0·3
XXVII.	+ 9	+ 169	+ ·19	+ 1·7
XXVIII.	+ 6	+ 176½	+ ·09	+ 0·5
XXIX.	- 19	+ 184	- ·07	+ 1·3
<hr/>				
XXXII.	- 8	+ 206½	- ·45	+ 3·6
XXXIII.	- 27	+ 214	- ·56	+ 15·1
XXXIV.	- 13	+ 221½	- ·66	+ 8·6
XXXV.	- 29	+ 229	- ·76	+ 22·0
Sum				<hr/> 118·9 <hr/>

28. The first column gives the sequence of periods observed by Baxendell. There is a wide gap between XI. and XXIV. with no observations, and it will be seen from columns 3 and 4 that it was unfortunately just in this gap that the corrections to epoch would have been largest, and given us an accurate value. Still, the periods when the correction vanishes are fairly well determined, which is something. They have been assumed to lie midway

between IV. and V., and between XXVIII. and XXIX. This is only a rough assumption, but the material does not warrant refinements. The "observed correction to epoch" given in the second column had been deduced from the observations by Miss Blagg, using a purely numerical process,* before there was any idea of applying them in this way. In the next three columns the product by $\sin(\arg)$ is deduced, and is almost uniformly positive. Dividing the sum 118.3 by $\sum \sin^2(\arg)$, which comes out 4.18, we get as above mentioned

$$C = 118.3 / 4.18 = 28.3.$$

S Serpentis (Ch. No. 5501).

29. For a different reason it is necessary to call attention to the case of *S Serpentis*, period 369 days. Chandler gives $C = 116$, $A = 4^\circ$, while our formulæ indicate $C = 30$, $A = 9^\circ$. But it is apparently possible to satisfy the observations in a different way, which assigns to C and A nearly the values of the formula. This investigation is given in a separate note following this paper.

30. What are now wanted are better determinations of C and A for stars with long and short periods. Too much depends on *S Cassiopeiæ* (610 days) and *R Virginis* (145 days), in the present state of our knowledge of them. "The dog is wagged by the tail." But it is the special value of an investigation of this kind that attention is directed to special needs which may guide observers in selection. The following stars, for instance, seem worth special attention from the northern observers:—

Short Periods.			Long Periods.		
Name.	No.	P	Name.	No.	P
Z Aquilæ	7260	127 ^d	S Cassiop.	432	610 ^d
W Cygni	7754	131	V Delphini	7458	540
R Vulpec.	7560	137	S Cephei	7779	486
R Virginis	4521	145	W Aquilæ	6900	480
S Aquilæ	7242	147	U Cygni	7299	461
V Capric.	7571	157	Z Sagittarii	6923	452
T Herculis	6512	165	R Leporis	1771	436
R Ceti	845	167	T Draco	6449	426
V Tauri	1717	170	R Cygni	7045	426
R Arietis	782	187	R Hydræ	4826	425

31. Returning now to Table I., in the seventh column is given $M - m$, the interval between maximum and the preceding minimum according to Chandler's revision. For most variables this is less than half the period, so that the quantity

$$aP = 2(M - m) - P$$

* The process can readily be inferred from the description on pp. 66 to 68 of *Mem. R.A.S.*, vol. lv., though this description is adapted to a period, *i.e.* a difference of epochs, instead of to the epoch itself.

shown in the last column is negative. The quantity α was used in *M.N.*, lxvii. p. 350, in connection with the classification of these variables according to their light curves, and it is interesting to see whether the type of light curve is related to the quantities C and A we have been discussing. The value of r for C and αP comes out .011, showing that there is complete independence. In considering the type of light curve it was early noticed that it seemed to be independent of the period; and it is not surprising therefore that C and A , which seem to depend on the period, should be independent of the type of curve. The relation between A and αP , and those between α alone and C and A , might be actually worked out, but it seems improbable that they will yield any result of interest.

32. Column 6 of Table I., headed "Phase," represents a much more speculative investigation, but one which it seemed just worth undertaking. It seemed quite possible that C and A , or at any rate A , might be *directly proportional to P* . The limits of possible error are such that there is nothing extravagant in this supposition, which would be represented by the equation

$$(A - 8^{\circ}.2) = k \left(\frac{P - 311}{50} \right),$$

where

$$k = \frac{8.2 \times 50}{311} = 1.32,$$

so that

$$A = .0264P.$$

We may compare the resulting values of A with those of § 15.

	$P = 100^d$	200^d	300^d	400^d	500^d	600^d
A by old equation =	4.4	6.2	8.0	9.8	11.6	13.4
A by new equation =	2.6	5.2	7.8	10.4	13.0	15.6

33. Now what makes this hypothesis of special interest is the fact that the cycle in which the term $C \sin(A^\circ E + \text{const.})$ is completed is $360/A$ periods or $360P/A$ days, and if P/A is constant we are confronted with the idea of a universal cycle controlling all these stars! The notion seems impossible, and yet there might be a simple explanation of it—the cause might be rooted in us, and not in the stars. It is difficult to see how any feature of the Earth or of the solar system could affect differential comparisons of distant stars, but though difficult it may not be impossible. The period indicated is $360/.0264$ days or 37.4 years. If there is a phenomenon of this kind originating with the observer, it might have the same phase for all stars, or at least the phase might depend on the star's position. Hence the phases given by Chandler were reduced to the common epoch 240 5000, and are shown in the sixth column of Table I. They certainly tend to no one value more than another, and a few experiments on their being related to a star's position gave no encouragement.

34. Another point may be mentioned. Whether there is

actually a single periodicity or not, the cycle for the periodic term is about of the same order for different stars. The value of $360P/A$ in years given by the equation of § 15 is

$P = 100^d$	200^d	300^d	400^d	500^d	600^d
Cycle = 22^y	32^y	37^y	40^y	43^y	46^y

Now if its coefficient also remains of the same order of magnitude, the maximum gradient of the disturbing cause (whatever it may be) may be nearly constant. Suppose it constant for simplicity. Then the displacement of maximum of a variable is due to the addition to the term

$$y_1 = f \cos 2\pi t/P$$

(which represents the ordinary variation near maximum) of a term

$$y_2 = kt,$$

where k may be nearly the same for all stars. The time of maximum will now be when

$$0 = \frac{dy_1}{dt} + \frac{dy_2}{dt} = -\frac{2\pi}{P} f \cdot \sin \frac{2\pi t}{P} + k,$$

or, since t is small,

$$t = \frac{kP^2}{4\pi^2 f}$$

Now f represents the range of variation, and is not very different for the stars in Table I., being usually about 5 magnitudes. For some stars the range is as low as 3, and for others it is as great as 7; and in a complete investigation the influence of this range must undoubtedly be considered. But for the present we will consider f constant. It follows that C , which is the maximum alteration of epoch, would be proportional to P^2 rather than to P . Hence it is worth inquiring whether the correlation of \sqrt{C} with P is better than that of C with P . On working it out, the result came $r = 0.34$, which is indeed greater than the $r = 0.30$ previously found, but not much. It would seem that for the present our best work can be done in getting improved values of C and A for individual stars, especially those above given, both by making new observations and by collecting and carefully discussing those already made.

SUMMARY.

§§ 1-3. Introductory.

§§ 4-5. Tabulation of Chandler's periodic "inequalities."

§§ 6-17. An example of working out the "correlation" r between two quantities A and P . The result found is $r = 0.56 \pm 0.08$, which is worth serious attention. The inference is that the argument A° in Chandler's periodic inequalities can be deduced from the period P days by the following probable formula:—

$$A = 8^\circ.2 + 0.18P.$$

§ 13 includes a table (III.) for the probable error of r .

§ 18. For C (the coefficient of the periodic inequalities) and P we find $r = 0.30 \pm 0.10$, which is not so conclusive. The relation would be

$$C = 7.3 + 0.64P.$$

§§ 19–29. Examination of 8 stars in detail, for which special information was available. The result is favourable. Remarks on S Serpentis, which seems to be exceptional.

§ 30. List of stars needing special attention.

§ 31. The type of light curve seems to be independent of C and A .

§§ 32–34. Possibility of a single periodic cause affecting all stars considered, but not supported.

Note on the Period of S Serpentis. By H. H. Turner, D.Sc.,
F.R.S., Savilian Professor.

1. The formula given for the maxima of S Serpentis (No. 5501) by Chandler in his Revision of the 3rd Catalogue (*A.J.*, No. 553) is

$$2388724 + 368.5 E + 116 \sin(4^\circ E + 62^\circ).$$

The periodic term in this formula attracted attention by considerable divergence from the value suggested by the preceding paper. The coefficient 116 days is much too large, and the argument $4^\circ E$ is too small. With a view to seeing whether it was well established, or whether perhaps some other formula would fit the observations, inquiry was made of Professor Müller, of Potsdam, who very kindly sent a complete list of observed maxima, with full references; adding a comparison with Chandler's formula, the remark that it did not fit more recent observations, and a suggestion of his own for improving it, which modified both the coefficient 116 and the argument $4^\circ E$ in the right direction. But this suggestion does not fit the observation of Lalande, and reasons will be given below why it is probably too early to suggest a completely satisfactory formula, in spite of the fact that Lalande's observation was made in 1794. Hence it seems unnecessary to reproduce here the full details, which will doubtless appear in the great work of reference for variable stars now being prepared by the German Committee of which Dr. Müller is a member.

2. For our present purpose the observations are sufficiently represented by the dates for every fifth maximum shown in the second column of Table I. The numeration in the first column is that of Chandler. In the third column are given the intervals, and we have to decide how to interpret Lalande's observation, which may belong to any one of the epochs -35 , -34 , or -33 . Chandler takes -33 ; and this gives an average interval of 1876 days for 5 periods, extending over 7×5 periods in all. This was quite a possible interpretation before the modern observations (represented